Topological shape optimization of permanent magnet of voice coil motor using level set method

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Abstract — In this paper, permanent magnet (PM) in voice coil motor is optimized by topological shape optimization using a level set method. The design velocities, normal to a moving boundary, are derived by shape sensitivity analysis using an adjoint variable method (AVM). The Lorentz force in the moving coil is maximized by means of the optimal distribution of the PM and the steel in the design domain.

I. INTRODUCTION

The proper use of a permanent magnet (PM) is quite required to enhance the performance of the electric devices. However, the conceptual design for positioning PM in an appliance is not a simple task.

The optimization using the level set method has been widely studied to overcome typical difficulties occurring in the topology optimization [1, 2]. So far, the PM optimization using the level set method has not been studied.

In this paper, the level set based optimization of the PM in the voice coil motor is proposed to maximize the Lorentz force on the moving coil.

II. LEVEL SET METHOD

Level set method is the numerical technique for tracing the moving boundary. Based on the boundary, the domain is divided into three parts of following Eq. (1) in accordance with the sign of the level set function Φ :

$$\begin{aligned}&\Phi(\mathbf{X}_\tau,\tau)>0,\ \forall \mathbf{X}_\tau\in\Omega\ \ \text{(material existence)},\\ &\Phi(\mathbf{X}_\tau,\tau)=0,\ \forall \mathbf{X}_\tau\in\Gamma^{\Phi}\ \ \text{(boundary)},\\ &\Phi(\mathbf{X}_\tau,\tau)<0,\ \forall \mathbf{X}_\tau\in\Omega_0\ \ \text{(void)},\end{aligned}\tag{1}$$

where $\mathbf{X}_\tau$ is geometric information at pseudo time τ .

The Hamilton-Jacobi partial differential equation (PDE) is derived by differentiating $\Phi(\mathbf{X}_\tau,\tau)$ in the boundary with respect to time τ and then applying the chain rule:

$$\begin{aligned}&\frac{\partial\Phi(\mathbf{X}_\tau,\tau)}{\partial\tau}+\frac{\partial\Phi(\mathbf{X}_\tau,\tau)}{\partial\mathbf{X}_\tau}\frac{d\mathbf{X}_\tau}{d\tau}=0,\\ &\text{where } \frac{\partial\Phi(\mathbf{X}_\tau,\tau)}{\partial\mathbf{X}_\tau}=\nabla\Phi,\ \ \frac{d\mathbf{X}_\tau}{d\tau}=V.\end{aligned}\tag{2}$$

The normal vector is given by:

$$\mathbf{n}=\nabla\Phi/\left|\nabla\Phi\right|.\tag{3}$$

The evolution equation of the level set function is obtained by substituting Eq. (3) into Eq. (2) as follows:

$$\frac{\partial\Phi(\mathbf{X}_\tau,\tau)}{\partial\tau}+V^n\left|\nabla\Phi\right|=0,\ \text{where } V^n=V\cdot\mathbf{n},\tag{4}$$

where V^n is the speed function in the moving boundary. The motion in the moving boundary is determined by Eq. (4) of Hamilton-Jacobi equation [3].

III. VARIATIONAL EQUATION IN MAGNETOSTATIC FIELD

A single governing equation in magnetostatic field is derived by a set of Maxwell's equations as follows:

$$\nabla\times(\frac{1}{\mu}\nabla\times\mathbf{A})=\mathbf{J}_s+\nabla\times(\frac{\mu_0}{\mu}\mathbf{M}).\tag{5}$$

where μ , $\mathbf{A}$, $\mathbf{J}_s$, μ_0 , and $\mathbf{M}$ are the material permeability, the magnetic vector potential, applied current density vector, the permeability in free space, and the remanent magnetization vector, respectively.

Multiplying the both sides of Eq. (5) with virtual vector potential $\bar{\mathbf{A}}$, and integrating over the analysis domain Λ :

$$\begin{aligned}&\iint_\Lambda\left[\nabla\times(\frac{1}{\mu}\nabla\times\mathbf{A})\right]\cdot\bar{\mathbf{A}}d\Lambda\\ &=\iint_\Lambda\left[\mathbf{J}_s+\nabla\times\frac{\mu_0}{\mu}\mathbf{M}\right]\cdot\bar{\mathbf{A}}d\Lambda,\ \text{for all } \bar{\mathbf{A}}\in\tilde{\mathbf{A}},\end{aligned}\tag{6}$$

where $\tilde{\mathbf{A}}$ is the space of admissible vector potential.

Applying Dirichlet and Neumann boundary conditions to Eq. (6), Eq. (7) is obtained [4]:

$$\begin{aligned}&\iint_\Lambda\left[(\nabla\times\mathbf{A})\cdot(\frac{1}{\mu}\nabla\times\bar{\mathbf{A}})\right]d\Lambda\\ &=\iint_\Lambda\left[\mathbf{J}_s\cdot\bar{\mathbf{A}}+\frac{\mu_0}{\mu}\mathbf{M}\cdot(\nabla\times\bar{\mathbf{A}})\right]d\Lambda.\end{aligned}\tag{7}$$

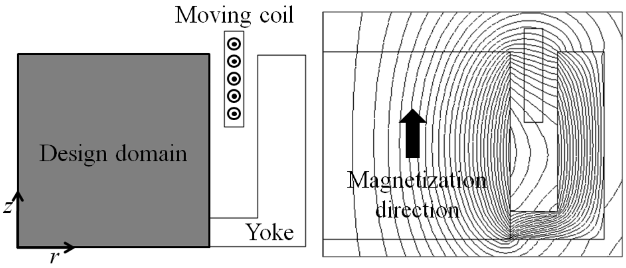
where the left side of Eq. (7) is an energy bilinear form $a(\mathbf{A},\bar{\mathbf{A}})$, and the right hand side is a load linear form $l(\bar{\mathbf{A}})$.

IV. LEVEL SET BASED OPTIMIZATION

The optimization problem takes the form

$$\begin{aligned}&\text{Minimize}\quad f_\Phi(\mathbf{A};\Phi)=\iint_{\Omega_{Obj}}g(\mathbf{A},\nabla\mathbf{A};\Phi)d\Omega,\\ &\text{subject to}\quad h=\mathbf{V}(\Phi)/\mathbf{V}_{\max}-1\le 0,\end{aligned}\tag{8}$$

where Ω_{Obj} , $\mathbf{V}$ and $\mathbf{V}_{\max}$ are the objective domain, the volume of the design domain and the maximum allowable volume, respectively.



(a) (b)

Fig. 1. Voice coil motor. (a) Initial design; (b) magnetic flux line.

The adjoint variable method (AVM) is used to calculate the design sensitivity. The Fréchet derivatives of objective function with respective to τ in the direction of the normal velocity are described as:

$$f' = -a'_{\delta\Phi}(\mathbf{A},\lambda;\Phi) + l'_{\delta\Phi}(\lambda;\Phi) + \iint_{\Omega_D} g_{,\Phi}\delta(\Phi)d\Omega, \quad (9)$$

where λ , Ω_D are the adjoint variable, and the design domain, respectively. λ is the response of the adjoint equation:

$$\iint_{\Lambda}\left[\nabla\times(\frac{1}{\mu}\nabla\times\lambda)\right]H(\Phi)d\Lambda = \iint_{\Lambda}\left[\mathbf{J}_{eq}\right]H(\Phi)d\Lambda. \quad (10)$$

where $\mathbf{J}_{eq}$ is the equivalent adjoint load.

A constrained optimization can be transformed to an unconstrained equation by using a Lagrange multiplier u:

$$L(\mathbf{A};\Phi,u,s) = \iint_{\Omega_{Obj}} g(\mathbf{A},\nabla\mathbf{A};\Phi)d\Omega + u(h+s^2), \quad (11)$$

where s is the slack variable converting an inequality constraint of Eq. (8) to an equality, and $u \geq 0$ must be satisfied from the Kuhn-Tucker condition.

The steepest descent direction is the derivative of Eq. (11) with respect to Φ :

$$\iint_{\Omega_D} V^n d\Omega = -\iint_{\Omega_D}\left(\mathbf{\Theta}(\mathbf{A},\lambda)+u\right)\delta\left(\Phi\right)d\Omega, \quad (12)$$

where $\mathbf{\Theta}(\mathbf{A},\lambda)$=

$$g_{,\Phi} - (\nabla\times\mathbf{A})\cdot(\frac{1}{\mu}\nabla\times\lambda) + \mathbf{J}_s\cdot\lambda + \frac{\mu_0}{\mu}\mathbf{M}\cdot(\nabla\times\lambda).$$

V. Optimization of PM in Voice Coil Motor

Fig. 1. (a) shows the cross section of a cylindrical voice coil motor, which is axisymmetric around the z-axis. The design domain, Ω_D, is initially magnetized to the direction of the z-axis, and the applied current on the moving coil, Ω_{Obj} , is 1 (A). The Lorentz force on Ω_{Obj} of the initial design is 3.12 (N).

The optimization problem is defined as follows:

$$\begin{aligned} &\text{Maximize} \quad f_{\Phi} = Lorentz\ force\big|_{\Omega_{Obj}}, \\ &\text{subject to} \quad h = \mathbf{V}(\Phi)/\mathbf{V}_{\max} - 1 \leq 0, \end{aligned} \quad (13)$$

where the volume of the PM on Ω_D is constrained by 70 (%) of the initial design, and the objective is to maximize the Lorentz force on the moving coil. For the material update of the optimization process, the PM is determined by $\Phi > 0$ in Eq. (1). Otherwise, it is the steel when Φ is negative.

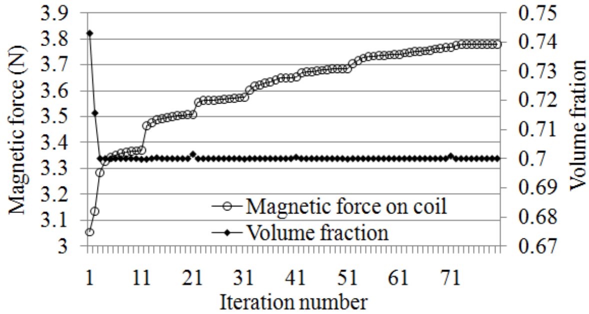
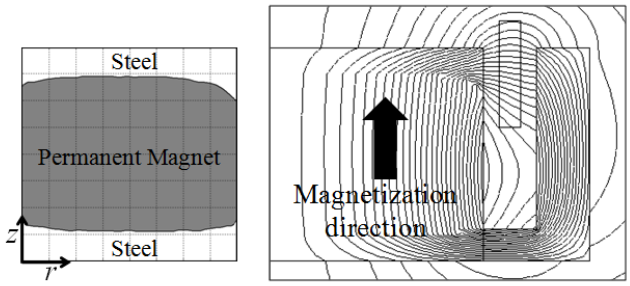


Fig. 2. Optimization history



(a) (b)

Fig. 3. Optimal design of PM. (a) Optimal level set design; (b) magnetic flux line.

TABLE I
COMPARISON BETWEEN INITIAL AND OPTIMAL DESIGN

	Initial Design	Optimal Design
Volume of PM [%]	100	70
Volume of Steel [%]	0	30
$Lorentz\ force\big\|_{\Omega_{Obj}}$ [%]	100	121.15

Fig. 3. (a) shows the optimal material distribution of the PM and the steel, and the Lorentz force on Ω_{Obj} is 3.78 (N). The optimal results are summarized in Table 1.

VI. Conclusion

In this paper, the level set based optimization of the PM is presented. The Lorentz force on the moving coil of a voice coil motor is maximized by the optimal material distribution of the PM and the steel.

VII. References

[1] G. Allaire, F. Jouve, and A. M. Toader, "Structural optimization using sensitivity analysis and a level-set method," *Journal of Computational Physics*, vol. 194, pp. 363-393, 2004.

[2] H. Shim, T. H. Vinh Thuy, S. Wang, and D. A. Tortorelli, "Topological shape optimization of electromagnetic problems using level set method and radial basis function," *CMES: Computer Modeling in Engineering & Science*, vol. 37, No. 2, pp. 175-202, 2008.

[3] T. J. Barth, and J. A. Sethian, "Numerical schemes for the Hamilton-Jacobi and level set equations on triangulated domains," *Journal of Computational Physics*, vol. 145, pp. 1-40, 1998.

[4] S. Wang, S. Jeong, and H. Yoon, "Continuum shape design sensitivity analysis of magnetostatic field using finite element method," *IEEE Trans. Magn*., vol. 35, No. 3. May 1999.