Topological shape optimization of permanent magnet of voice coil motor using level set method

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Abstract — In this paper, permanent magnet (PM) in voice coil motor is optimized by topological shape optimization using a level set method. The design velocities, normal to a moving boundary, are derived by shape sensitivity analysis using an adjoint variable method (AVM). The Lorentz force in the moving coil is maximized by means of the optimal distribution of the PM and the steel in the design domain.

I. INTRODUCTION

The proper use of a permanent magnet (PM) is quite required to enhance the performance of the electric devices. However, the conceptual design for positioning PM in an appliance is not a simple task.

The optimization using the level set method has been widely studied to overcome typical difficulties occurring in the topology optimization [1, 2]. So far, the PM optimization using the level set method has not been studied.

In this paper, the level set based optimization of the PM in the voice coil motor is proposed to maximize the Lorentz force on the moving coil.

II. LEVEL SET METHOD

Level set method is the numerical technique for tracing the moving boundary. Based on the boundary, the domain is divided into three parts of following Eq. (1) in accordance with the sign of the level set function Φ : 1. INTRODUCTION

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\Phi(\mathbf{X}_r, \tau) > 0, \ \forall \mathbf{X}_r \in \Omega \ \text{ (material existence)},
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\Phi(\mathbf{X}_r, \tau) = 0, \ \forall \mathbf{X}_r \in \Gamma^{\Phi} \ \text{(boundary)}, \tag{1}
$$
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\Phi(\mathbf{X}_r, \tau) < 0, \ \forall \mathbf{X}_r \in \Omega_c \ \text{(void)}.
$$

where X_{τ} is geometric information at pseudo time τ .

The Hamilton-Jacobi partial differential equation (PDE) respect to time τ and then applying the chain rule:

using the level set method has not been studied.
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\ncoil motor is proposed to maximize the Lorentz
\nmoving coil.
\nII. LEVEL SET METHOD
\nthe level set of following Eq. (1) in accordance
\nthe level set function Φ:
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= \int_{\Lambda} \left[\nabla \times (\frac{1}{\mu} \nabla \times \mathbf{\bar{A}}) \right] \cdot \mathbf{\bar{A}} d\Lambda
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\nwhere $\mathbf{\tilde{A}} \text{ is the space of admissible vector}$
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= \int_{\Lambda} \left[\nabla \times \mathbf{A} \cdot \frac{\mu_0}{\mu} \mathbf{M} \right] \cdot \mathbf{\bar{A}} d\Lambda
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\ngometric information at pseudo time τ .
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= \int_{\Lambda} \left[\nabla \times \mathbf{A} \cdot \frac{\mu_0}{\mu} \mathbf{M} \cdot (\nabla \times \mathbf{\bar{A}}) \right] d\Lambda
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$$
= \int_{\Lambda} \left[\nabla \times \mathbf{A} \cdot \frac{\mu_0}{\mu} \mathbf{M} \cdot (\
$$

The normal vector is given by:

$$
\mathbf{n} = \nabla \Phi / |\nabla \Phi|.
$$
 (3)

The evolution equation of the level set function is obtained by substituting Eq. (3) into Eq. (2) as follows:

$$
\frac{\partial \Phi(\mathbf{X}_{\tau}, \tau)}{\partial \tau} + V^n |\nabla \Phi| = 0, \text{ where } V^n = V \cdot \mathbf{n}, \quad (4)
$$

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 $\frac{\partial \Phi(\mathbf{X}_r, \tau)}{\partial \tau} + V^n |\nabla \Phi| = 0$, where $V^n = V \cdot \mathbf{n}$, (4)
 V^n is the speed function in the moving boundary.

tion in the moving bou where V^n is the speed function in the moving boundary.
The motion in the moving boundary is determined by Eq. (4) of Hamilton-Jacobi equation [3].

III. VARIATIONAL EQUATION IN MAGNETOSTATIC FIELD

A single governing equation in magnetostatic field is derived by a set of Maxwell's equations as follows:

$$
\nabla \times (\frac{1}{\mu} \nabla \times \mathbf{A}) = \mathbf{J}_s + \nabla \times (\frac{\mu_0}{\mu} \mathbf{M}).
$$
 (5)

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Example 20. Where $V^n = V \cdot \mathbf{n}$, (4)

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Multiplying the both sides of Eq. (5) with virtual vector potential \overline{A} , and integrating over the analysis domain Λ :

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where \tilde{A} is the space of admissible vector potential.

Applying Dirichlet and Neumann boundary conditions to Eq. (6), Eq. (7) is obtained [4]:

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\nthe numerical result, the magnetic device. μ , A, J, J, U_0 (5)
\nusing the level set method has been
\ncone typical difficulties occurring in
\n2ation [1, 2]. So far, the PM
\nlevel set based optimization of the PM
\nseted set method has not been studied.
\n
$$
= \iint_{\Delta} \left[J_x + \nabla \times \frac{L}{\mu} \nabla \times \frac{
$$

where the left side of Eq. (7) is an energy bilinear form

IV. LEVEL SET BASED OPTIMIZATION

 $\frac{dX_t}{dt} = V$. The optimization problem takes the form

Minimize
$$
f_{\Phi}(\mathbf{A}; \Phi) = \iint_{\Omega_{\text{Obj}}} g(\mathbf{A}, \nabla \mathbf{A}; \Phi) d\Omega,
$$

subject to $h = \mathbf{V}(\Phi) / \mathbf{V}_{\text{max}} - 1 \le 0,$ (8)

where Ω_{Obj} , **V** and V_{max} are the objective domain, the volume of the design domain and the maximum allowable volume, respectively.

Fig. 1. Voice coil motor. (a) Initial design; (b) magnetic flux line.

The adjoint variable method (AVM) is used to calculate the design sensitivity. The Fréchet derivatives of objective function with respective to τ in the direction of the normal velocity are described as:

$$
f' = -a'_{\infty}(\mathbf{A}, \lambda; \Phi) + l'_{\infty}(\lambda; \Phi) + \iint_{\Omega_D} g_{,\Phi} \delta(\Phi) d\Omega, \quad (9)
$$

where λ , Ω _{*D*} are the adjoint variable, and the design domain, respectively. λ is the response of the adjoint equation: where λ , Ω_{ρ} are the adjoint variable, and the design

equation:

equation:
 $\iint_{\lambda} \left[\nabla \times (\frac{1}{\mu} \nabla \times \lambda) \right] H(\Phi) d\Lambda = \iint_{\lambda} [\mathbf{J}_{\infty}] H(\Phi) d\Lambda$. (10)

where \mathbf{J}_{∞} is the equivalent adjoint load.

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$$
\iint_{\Lambda} \left[\nabla \times (\frac{1}{\mu} \nabla \times \lambda) \right] H(\Phi) d\Lambda = \iint_{\Lambda} \left[\mathbf{J}_{eq} \right] H(\Phi) d\Lambda. \quad (10)
$$

where J_{eq} is the equivalent adjoint load.

A constrained optimization can be transformed to an unconstrained equation by using a Lagrange multiplier *u*:

$$
L(\mathbf{A}; \Phi, u, s) = \iint_{\Omega_{Obj}} g(\mathbf{A}, \nabla \mathbf{A}; \Phi) d\Omega + u(h + s^2), \quad (11)
$$

where *s* is the slack variable converting an inequality constraint of Eq. (8) to an equality, and $u \ge 0$ must be satisfied from the Kuhn-Tucker condition. *D*,*u*,*s*) = $\iint_{\Omega_{O|b}} g(A, \nabla A; \Phi) d\Omega + u(h + s^2)$, (11

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The steepest descent direction is the derivative of Eq. (11) with respect to Φ :

$$
\iint_{\Omega_D} V^n d\Omega = -\iint_{\Omega_D} \left(\Theta(\mathbf{A}, \lambda) + u \right) \delta(\Phi) d\Omega, \tag{12}
$$

 λ)= where $\Theta(A, \lambda)$ =

$$
g_{,\Phi} - (\nabla \times \mathbf{A}) \cdot (\frac{1}{\mu} \nabla \times \lambda) + \mathbf{J}_s \cdot \lambda + \frac{\mu_0}{\mu} \mathbf{M} \cdot (\nabla \times \lambda).
$$

In this paper, th

V. OPTIMIZATION OF PM IN VOICE COIL MOTOR

Fig. 1. (a) shows the cross section of a cylindrical voice coil motor, which is axisymmetric around the *z*-axis. The design domain, Ω_D , is initially magnetized to the direction of the z-axis, and the applied current on the moving coil, Ω_{Obj} , is 1 (A). The Lorentz force on Ω_{Obj} of the initial design is 3.12 (N). of Fig. (8) to an equality, and $u \ge 0$ must be

subject to Φ -poly in Eq. (1). Otherwise, it is the stell when Φ

subspect to Φ -poly in Eq. (1). Otherwise, it is the stell when Φ

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Fig. 3.78 **Exertion is the derivative of Eq.** (1). Otherwise, it is the steel when $(\Theta(A, \lambda) + u)\delta(\Phi)d\Omega$, (12) The applies $\Gamma_{\rm IS}$. 3, (a) shows the optimal material distribution of $(\Theta(A, \lambda) + u)\delta(\Phi)d\Omega$, (12) PM and the steel, and the

The optimization problem is defined as follows:

$$
\begin{array}{ll}\n\text{Maximize} & f_{\Phi} = \text{Lorentz force} \big|_{\Omega_{\text{Obj}}}, & \text{Moadel} \\
\text{subject to} & h = \mathbf{V}(\Phi) / \mathbf{V}_{\text{max}} - 1 \le 0, & \text{[3]} \quad \text{T. J. B4} \\
\text{Jacobi} & \text{[3]} \quad \text{T. J. B5}\n\end{array}
$$

where the volume of the PM on Ω_D is constrained by 70 (%) of the initial design, and the objective is to maximize the Lorentz force on the moving coil. For the material

Fig. 3. Optimal design of PM. (a) Optimal level set design; (b) magnetic flux line.

update of the optimization process, the PM is determined by negative.

Fig. 3. (a) shows the optimal material distribution of the PM and the steel, and the Lorentz force on Ω_{Obs} is 3.78 (N). The optimal results are summarized in Table 1.

VI. CONCLUSION

In this paper, the level set based optimization of the PM is presented. The Lorentz force on the moving coil of a voice coil motor is maximized by the optimal material distribution of the PM and the steel.

VII. REFERENCES

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